Discrete Convolution			
Linear Convolution	$h[k] * x[k]$	$=$	$\sum\limits_{\kappa=-\infty}^{\infty} h[\kappa] x[k-\kappa]$
Periodic Convolution	$h[k] \circledast_N x[k]$	$=$	$\sum\limits_{\kappa=0}^{N-1} h[\kappa] x[(k-\kappa) \bmod N]$
Properties and Rules			
Commutativity	$x[k] * h[k]$	$=$	$h[k] * x[k]$
Associativity	$(x[k] * g[k]) * h[k]$	$=$	$x[k] * (g[k] * h[k])$
Distributivity	$x[k] * (g[k] + h[k])$	$=$	$(x[k] * g[k]) + (x[k] * h[k])$
Neural Element	$x[k] * \delta[k]$	$=$	$x[k]$
Multiplication	$a(x[k] * h[k])$	$=$	$a\, x[k] * h[k] = x[k] * a\, h[k]$

Discrete-Time Fourier Transform (DTFT)			
$X(e^{j\Omega}) = \mathcal{F}_* \{x[k]\} = \sum\limits_{k=-\infty}^{\infty} x[k] e^{-j\Omega k}$			$x[k] = \mathcal{F}_*^{-1} \left\{ X(e^{j\Omega}) \right\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega k} \mathrm{d}\Omega$
Properties and Theorems			
Periodicity	$X(e^{j\Omega})$	$=$	$X(e^{j(\Omega+2\pi)})$
Time Reversal	$x[-k]$	$\circ\!\!-\!\!\bullet$	$X(e^{-j\Omega})$
Conjugation	$x^*[k]$	$\circ\!\!-\!\!\bullet$	$X^*(e^{-j\Omega})$
Convolution	$x[k] * h[k]$	$\circ\!\!-\!\!\bullet$	$X(e^{j\Omega}) \cdot H(e^{j\Omega})$
Multiplication	$x[k] \cdot h[k]$	$\circ\!\!-\!\!\bullet$	$\frac{1}{2\pi} X(e^{j\Omega}) \circledast H(e^{j\Omega})$
Shift $(\kappa \in \mathbb{Z})$	$x[k-\kappa]$	$\circ\!\!-\!\!\bullet$	$e^{-j\Omega\kappa} X(e^{j\Omega})$
Modulation $(\Omega_0 \in \mathbb{R})$	$e^{j\Omega_0 k} x[k]$	$\circ\!\!-\!\!\bullet$	$X(e^{j(\Omega-\Omega_0)})$
Multiplication by k	$k x[k]$	$\circ\!\!-\!\!\bullet$	$j \frac{d}{d\Omega} X(e^{j\Omega})$
Parseval's Theorem	$\sum\limits_{k=-\infty}^{\infty} \lvert x[k] \rvert^2$	$=$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left\lvert X(e^{j\Omega}) \right\rvert^2 \mathrm{d}\Omega$
Correspondences			
	$\delta[k]$	$\circ\!\!-\!\!\bullet$	1
	1	$\circ\!\!-\!\!\bullet$	$\text{III}\left(\frac{\Omega}{2\pi}\right)$
	$\epsilon[k]$	$\circ\!\!-\!\!\bullet$	$\frac{1}{1-e^{-j\Omega}} + \frac{1}{2}\text{III}\left(\frac{\Omega}{2\pi}\right)$
	$e^{j\Omega_0 k}$	$\circ\!\!-\!\!\bullet$	$\text{III}\left(\frac{\Omega-\Omega_0}{2\pi}\right)$
	(für $\lvert a \rvert < 1$) $a^k \epsilon[k]$	$\circ\!\!-\!\!\bullet$	$\frac{1}{1-a e^{-j\Omega}}$
	$\cos[\Omega_0 k]$	$\circ\!\!-\!\!\bullet$	$\frac{1}{2}\left(\text{III}\left(\frac{\Omega+\Omega_0}{2\pi}\right) + \text{III}\left(\frac{\Omega-\Omega_0}{2\pi}\right)\right)$
	$\sin[\Omega_0 k]$	$\circ\!\!-\!\!\bullet$	$\frac{j}{2}\left(\text{III}\left(\frac{\Omega+\Omega_0}{2\pi}\right) - \text{III}\left(\frac{\Omega-\Omega_0}{2\pi}\right)\right)$

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Discrete Fourier Transform (DFT)

$$X[\mu] = \mathrm{DFT}_N\left\{x[k]\right\} = \sum_{k=0}^{N-1} x[k] e^{-j\frac{2\pi}{N}\mu k} \qquad x[k] = \mathrm{IDFT}_N\left\{X[\mu]\right\} = \frac{1}{N}\sum_{\mu=0}^{N-1} X[\mu] e^{j\frac{2\pi}{N}\mu k}$$

Properties and Theorems

Periodicity	$X[\mu]$	$=$	$X[\mu+N]$
	$x[k]$	$=$	$x[k+N]$
Time Reversal	$x[-k]$	$\circ\!-\!\bullet$	$X[N-\mu]$
Conjugation	$x^*[k]$	$\circ\!-\!\bullet$	$X^*[N-\mu]$
Periodic Convolution	$x[k] \circledast h[k]$	$\circ\!-\!\bullet$	$X[\mu]\cdot H[\mu]$
Multiplication	$x[k]\cdot h[k]$	$\circ\!-\!\bullet$	$\frac{1}{N} X[\mu] \circledast H[\mu]$
Cyclic Shift $(\kappa \in \mathbb{Z})$	$x[k-\kappa]$	$\circ\!-\!\bullet$	$e^{-j\frac{2\pi}{N}\mu\kappa} X[\mu]$
Modulation $(\lambda \in \mathbb{Z})$	$e^{j\frac{2\pi}{N}k\lambda} x[k]$	$\circ\!-\!\bullet$	$X[\mu-\lambda]$
Parseval's Theorem	$\sum_{k=0}^{N-1} \lvert x[k]\rvert^2$	$=$	$\frac{1}{N}\sum_{\mu=0}^{N-1} \lvert X[\mu]\rvert^2$

Correspondences

$\delta[k]$	$\circ\!-\!\bullet$	1
1	$\circ\!-\!\bullet$	$N\cdot\delta[\mu]$
$e^{j\Omega_0 k}$	$\circ\!-\!\bullet$	$e^{j(\Omega_0-\mu\frac{2\pi}{N})\frac{N-1}{2}} \cdot \frac{\sin\left[\frac{N(\Omega_0-\mu\frac{2\pi}{N})}{2}\right]}{\sin\left[\frac{\Omega_0-\mu\frac{2\pi}{N}}{2}\right]}$
$\mathrm{rect}_M[k]$	$\circ\!-\!\bullet$	$e^{-j\pi\mu\frac{M-1}{N}} \cdot \frac{\sin\left[\frac{M\pi\mu}{N}\right]}{\sin\left[\frac{\pi\mu}{N}\right]}$

z-Transform

$$X(z) = \mathcal{Z}\left\{x[k]\right\} = \sum_{k=-\infty}^{\infty} x[k] z^{-k} \qquad x[k] = \mathcal{Z}^{-1}\left\{X(z)\right\} = \frac{1}{2\pi j}\oint_{C\subset \mathrm{Kb}} X(z) z^{k-1} \mathrm{d}z$$

Properties and Theorems

				Convergence
Linearity	$A\,x_1[k] + B\,x_2[k]$	$\circ\!-\!\bullet$	$A\,X_1(z) + B\,X_2(z)$	$\mathrm{Kb} \supseteq \mathrm{Kb}\{X_1\} \cap \mathrm{Kb}\{X_2\}$
Time Reversal	$x[-k]$	$\circ\!-\!\bullet$	$X(z^{-1})$	$\left\{z \mid z^{-1} \in \mathrm{Kb}\{X\}\right\}$
Conjugation	$x^*[k]$	$\circ\!-\!\bullet$	$X^*(z^*)$	$\mathrm{Kb}\{X\}$
Shift $(\kappa \in \mathbb{Z})$	$x[k-\kappa]$	$\circ\!-\!\bullet$	$z^{-\kappa} X(z)$	$\mathrm{Kb}\{X\}$ } $z=0$ und $z\to\infty$ have to be considered separately
Multiplication by k	$k x[k]$	$\circ\!-\!\bullet$	$-z\frac{d}{dz} X(z)$	$\mathrm{Kb}\{X\}$ }
Modulation $(a \in \mathbb{C})$	$a^k x[k]$	$\circ\!-\!\bullet$	$X(\frac{z}{a})$	$\left\{z \mid \frac{z}{a} \in \mathrm{Kb}\{X\}\right\}$
Convolution	$x[k] * h[k]$	$\circ\!-\!\bullet$	$X(z)\cdot H(z)$	$\mathrm{Kb} \supseteq \mathrm{Kb}\{X_1\} \cap \mathrm{Kb}\{H\}$

Correspondences

$\delta[k]$	$\circ\!-\!\bullet$	1	$\mathbb{C}$
$\epsilon[k]$	$\circ\!-\!\bullet$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$a^k \epsilon[k]$	$\circ\!-\!\bullet$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$-a^k \epsilon[-k-1]$	$\circ\!-\!\bullet$	$\frac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$
$k\epsilon[k]$	$\circ\!-\!\bullet$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
$k\, a^k \epsilon[k]$	$\circ\!-\!\bullet$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$\sin[\Omega_0 k]\epsilon[k]$	$\circ\!-\!\bullet$	$\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$
$\cos[\Omega_0 k]\epsilon[k]$	$\circ\!-\!\bullet$	$\frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$

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