Numerical Linear Algebra

Mid-term test

Fall 2015

Variant 2

Questions (40 pts)

Q1 1.1 What is the complexity of matrix-vector multiplication? (5 pts)

1.2 Is it possible to reduce this complexity for general matrix and vector? Why? (5 pts)

Q2 Suppose that you want to calculate the best low-rank approximation of a certain matrix with absolute precision ϵ. What should you do? How much memory do you need to store the full matrix and the rank-r approximation? (10 pts)

Q3 Is it a good idea to find eigenvalues of large matrices via characteristic polynomial? Why? (10 pts)

Q4 What is pseudoinverse of a matrix? How can it be calculated using the SVD? For what purposes it can be useful? (10 pts)

Problems (60 pts)

P1 Find $\|F_n\|_1$ and $\|F_n\|_F$, where F_n is the Fourier matrix of size $n \times n$. (10 pts)

P2 Find skeleton decomposition of a matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. (10 pts)

P3 Suppose you are given a linear model $y = ax + b$ and data points (x, y): $(0, 1)$, $(1, 2)$, $(2, 4)$. Write down a system on coefficients a and b and find its least squares solution. (10 pts)

P4 Show that any normal triangular matrix is diagonal. (10 pts)

P5 Find $\mathrm{cond}_2 \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix}$, where $\epsilon \neq 0$. Note: subscript 2 in cond_2 means that second norm is used. (10 pts)

P6 The goal of compressed sensing is to find the sparsest solution x of an undetermined linear system $y = Ax$ where $A \in \mathbb{R}^{n \times m}$, $n < m$. In order to achieve it one could try to find solution which has minimal first norm. Intuition behind this fact is quite simple in 2D:

6.1 Draw disks $\|x\| = \text{const}$ in $1, 2$ and ∞ norms. (5 pts)

6.2 Find graphically solutions of $y = Ax$, $\|x\|_* \to \min$, where $A \in \mathbb{R}^{1 \times 2}$ and $* = \{1, 2, \infty\}$. Which norm yields the sparsest solution? (5 pts)